Curved Beams

Circumferential Stresses Problems Solution

Mechanics of Solids-2

Lecture # 7

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Summary of Previous Lecture

- Derivation of Stresses in Curved Beam.

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- Problem Solving: To Find Circumferential Stresses in Curved Beams.

Curved Beams
Problem Solving

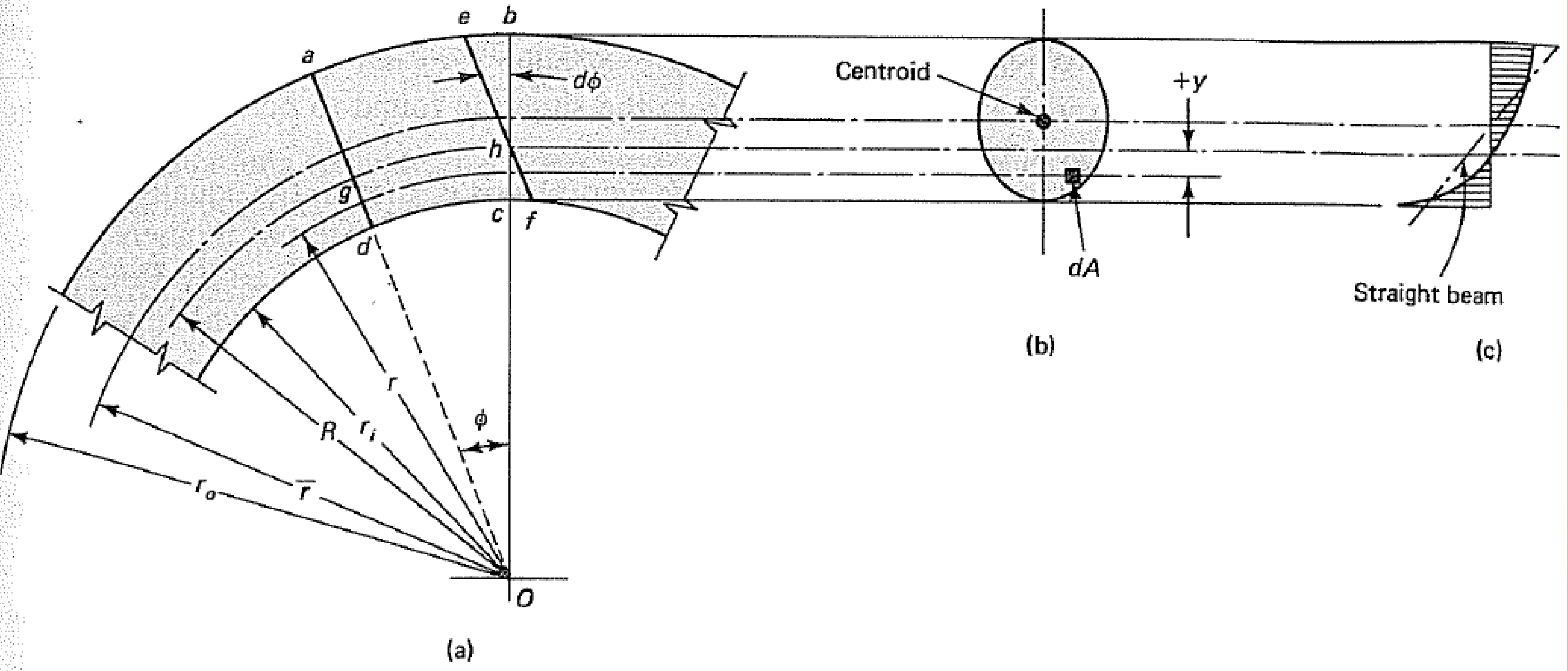


Fig. 6-23 Curved bar in pure bending.

Curved Beams
Problem Solving

$$R = \frac{A}{\int_A dA/r}$$

For calculation of Neutral Axis

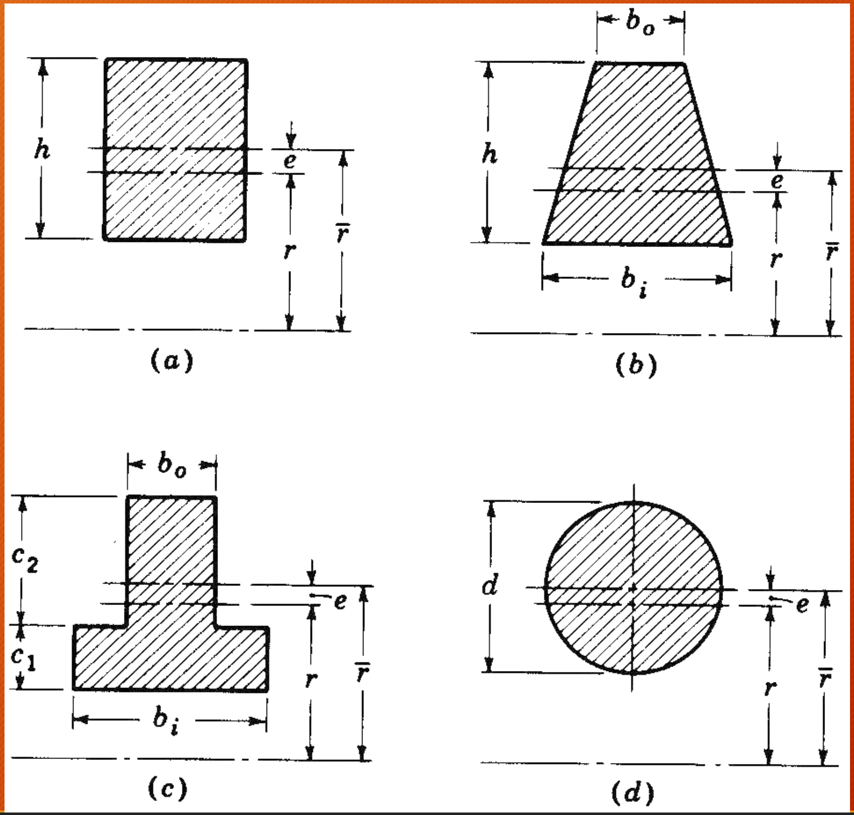
$$\sigma = \frac{M(R - r)}{rA(\bar{r} - R)}$$

For calculation of Stresses

$$\sigma_i = \frac{Mc_i}{Aer_i} \qquad \sigma_o = \frac{Mc_o}{Aer_o} \qquad (7)$$

Curved Beams - Derivation of Stress
Formulas for Some Common Sections

Sections most frequently encountered in the stress analysis of curved beams are shown below.



For the rectangular section shown in (a), the formulae are

$$\bar{r} = r_i + \frac{h}{2} \qquad and \qquad r = \frac{h}{\ln(r_o/r_i)}$$

For the trapezoidal section in (b), the formulae are

$$\bar{r} = r_i + \frac{h}{3}\frac{b_i + 2b_o}{b_i + b_o}$$

$$r = \frac{A}{b_o - b_i + \left[(b_i r_o - b_o r_i)/h\right]\ln(r_o/r_i)}$$

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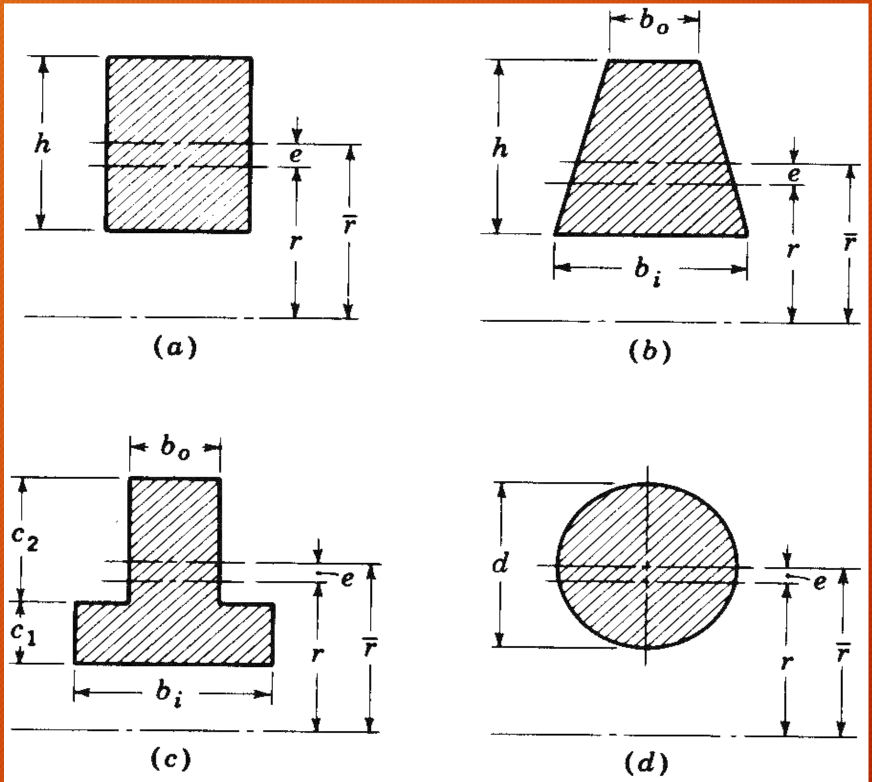
Curved Beams - Derivation of Stress

Formulas for Some Common Sections

Sections most frequently encountered in the stress analysis of curved beams are shown below.



For the T section in we have

$$\bar{r} = r_i + \frac{b_i c_1^2 + 2 b_o c_1 c_2 + b_o c_2^2}{2\left(b_o c_2 + b_i c_1\right)}$$

$$r = \frac{b_i c_1 + b_o c_2}{b_i \ln\left[\left(r_i + c_1\right)/ r_i\right] + b_o \ln\left[r_o /\left(r_i + c_1\right)\right]}$$

The equations for the solid round section of Fig. *(d)* are

$$\bar{r} = r_i + \frac{d}{2}$$

$$r = \frac{d^2}{4\left(2\bar{r} - \sqrt{4\bar{r}^2 - d^2}\right)}$$

Compare stresses in a 50 by 50 mm rectangular bar subjected to end moments of 2083 N·m in three special cases: (a) straight beam, (b) beam curved to a radius of 250 mm along the centroidal axis, i.e., $\bar{r} = 250$ mm, Fig. 6-24(a), and (c) beam curved to $\bar{r} = 75$ mm.

(*a*) This follows directly by applying Eqs. 6-21 and 6-22.

Part-A

$$S = bh^2/6 = 50 \times 50^2/6 = 20.83 \times 10^3 \text{ mm}^3$$

$$\sigma_{\max} = \frac{M}{S} = \frac{2083 \times 10^3}{20.83 \times 10^3} = \pm 100 \text{ MPa}$$

This result is shown in Fig. 6-24(c). $\bar{r} = \infty$ since a straight bar has an infinite radius of curvature.

To solve parts (*b*) and (*c*) the neutral axis must be located first. This is found in general terms by integrating Eq. 6-30. For the rectangular section, the elementary area is taken as $b\,dr$, Fig. 6-24(b). The integration is carried out between the limits r_i and r_o, the inner and outer radii, respectively.

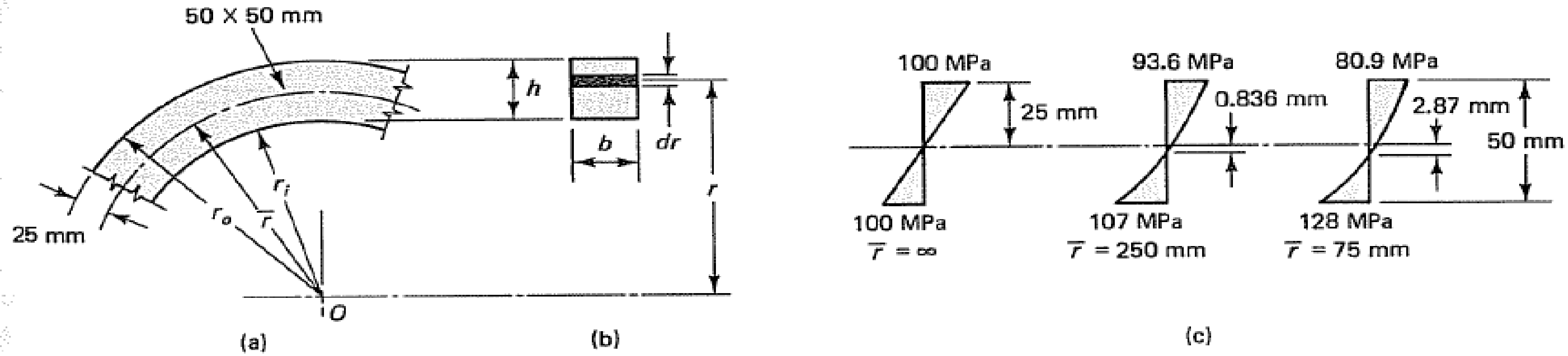


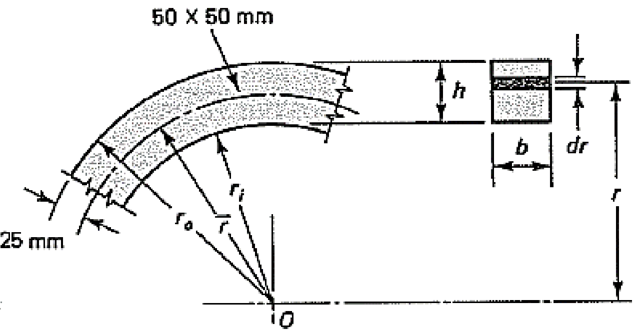
Fig. 6-24

Part-B

$$R = \frac{A}{\int_A dA/r} = \frac{bh}{\int_{r_i}^{r_o} b\, dr/r} = \frac{h}{\int_{r_i}^{r_o} dr/r}$$
$$= \frac{h}{|\ln r|_{r_i}^{r_o}} = \frac{h}{\ln (r_o/r_i)} = \frac{h}{2.3026 \log (r_o/r_i)} \qquad (6\text{-}33)$$

where h is the depth of the section, ln is the natural logarithm, and log is a logarithm with a base of 10 (common logarithm).

(b) For this case, $h = 50$ mm, $\bar{r} = 250$ mm, $r_i = 225$ mm, and $r_o = 275$ mm. The solution is obtained by evaluating Eqs. 6-33 and 6-31. Subscript i refers to the normal stress σ of the inside fibers; o of the outside fibers.



$$R = \frac{50}{\ln (275/225)} = 249.164 \text{ mm}$$
$$e = \bar{r} - R = 250 - 249.164 = 0.836 \text{ mm}$$
$$\sigma_i = \frac{M(R - r_i)}{r_i A(\bar{r} - R)} = \frac{2083 \times 10^3 \times (249.164 - 225)}{225 \times 50^2 \times 0.836}$$
$$= 107 \text{ MPa}$$

F

$$\sigma_o = \frac{M(R - r_o)}{r_o A(\bar{r} - R)} = \frac{2083 \times 10^3 \times (249.164 - 275)}{275 \times 50^2 \times 0.836}$$
$$= -93.6 \text{ MPa}$$

The negative sign of σ_o indicates a compressive stress. These quantities and the corresponding stress distribution are shown in Fig. 6-24(c); $\bar{r} = 250$ mm.

Part-C

(c) This case is computed in the same way. Here $h = 50$ mm, $\bar{r} = 75$ mm, $r_i =$ 50 mm, and $r_o = 100$. Results of the computation as shown in Fig. 6-24(c).

$$R = \frac{50}{\ln(100/50)} = \frac{50}{\ln 2} = 72.13 \text{ mm}$$

$$e = \bar{r} - R = 75 - 73.13 = 2.87 \text{ mm}$$

$$\sigma_i = \frac{2083 \times 10^3 \times (72.13 - 50)}{50 \times 50^2 \times 2.87} = 128 \text{ MPa}$$

$$\sigma_o = \frac{2083 \times 10^3 \times (72.13 - 100)}{100 \times 50^2 \times 2.87} = -80.9 \text{ MPa}$$

Curved Beams

Pro 3

Several important conclusions, generally true, may be reached from this example. First, *the usual flexure formula is reasonably good for beams of considerable curvature*. Only 7 percent error in the maximum stress occurs in part (b) for $\bar{r}/h = 5$, an error tolerable for most applications. For greater ratios of $\bar{r}/h$, this error diminishes. As the curvature of the beam increases, the stress on the concave side rapidly increases over the one given by the usual flexure formula. When $\bar{r}/h = 1.5$, a 28 percent error occurs. Second, the evaluation of the integral for R over the cross-sectional area may become very complex. Finally, calculations of R must be very accurate since differences between R and numerically comparable quantities are used in the stress formula.

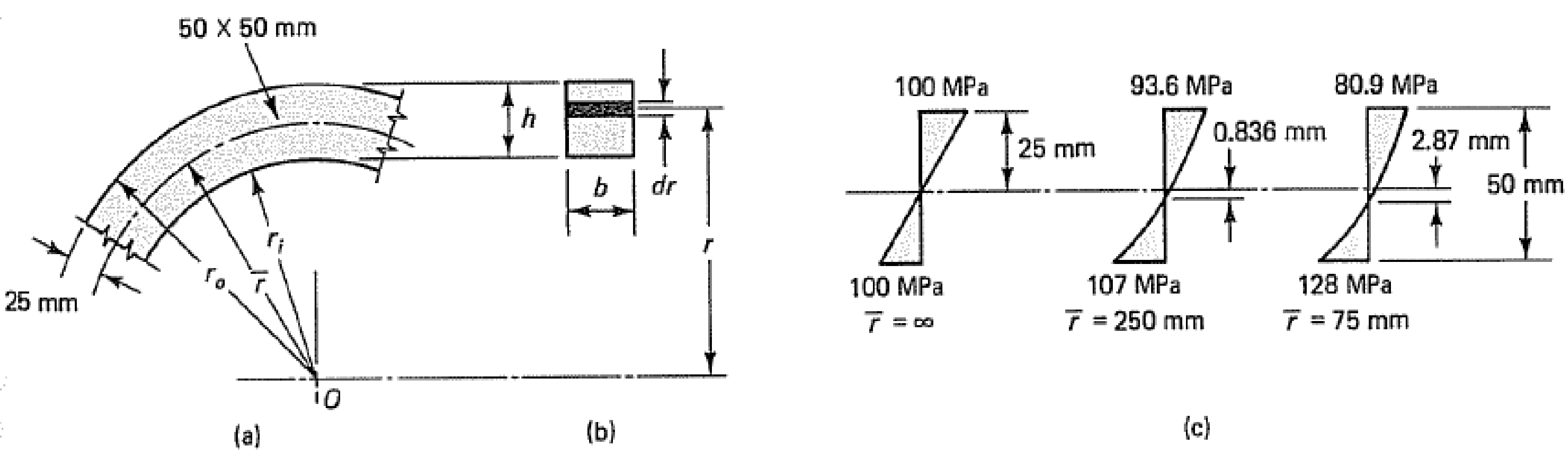


Fig. 6-24

Thank-you for Listening!

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Shad