# Joint Efficiency and Spherical Cylinders 

Mechanics of Solids-2

Lecture \# 3

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## Summary of Previous Lectures

- Thin and Thick Walled Cylinders
- Applications of Thin and Thick Walled Cylinders
- Three Different forms of Stresses in Cylinders
- Longitudinal Stresses
- Longitudinal Strains
- Change in Length
- Hoop or Circumferential Stresses
- Circumferential Strain
- Change in Diameter
- Change in Volume
- Radial Stresses
- Derivation of Formulae for Stresses in Thin and Thick Walled Cylinders
- Application of Longitudinal, Circumferential, and Radial Stresses on Real World Engineering Problems.


## Table of Content for Lecture \# 3

- Joint Efficiency in Cylinders
- Derivation of Formulae for Joint Efficiency in Cylinders
- Example Problems on Joint Efficiency.
- Spherical Pressure Vessels
- Problem on Spherical Pressure Vessels
- Few implications (deductions) on Cylindrical and Spherical Pressure Vessels.


## JOINT EFFICIENCY

Steel plates of only particular lengths and width are available. Hence whenever larger size cylinders (like boilers) are required, a number of plates are to be connected. This is achieved by using riveting in circumferential and longitudinal directions as shown in figure. Due to the holes for rivets, the net area of cross section decreases and hence the stresses increase.


## JOINT EFFICIENCY

The cylindrical shells like boilers are having two types of joints namely Longitudinal Joints and Circumferential Joints. Due to the holes for rivets, the net area of cross section decreases and hence the stresses increase. If the efficiencies of these joints are known, the stresses can be calculated as follows.

Let $\eta_{L}=$ Efficiency of Longitudinal joint and $\eta_{C}=$ Efficiency of Circumferential joint.

Circumferential Stress is given by,

A shell is a type of structural element which is characterized by its geometry, being a threedimensional solid whose thickness is very small when compared with other dimensions

Longitudinal stress is given by,


Note: In longitudinal joint, the circumferential stress is developed and in circumferential joint, longitudinal stress is developed.


If A is the gross area and $\mathrm{A}_{\text {eff }}$ is the effective resisting area then, Efficiency $=\mathrm{A}_{\text {eff }} / \mathrm{A}$

Bursting force $=p$ Ld
Resisting force $=\sigma_{c} \times A_{\text {eff }}=\sigma_{c} \times \eta_{L} \times A=\sigma c \times \eta_{L} \times 2 \mathrm{tL}$
Where $\eta_{\mathrm{L}}=$ Efficiency of Longitudinal joint
Bursting force $=$ Resisting force
$\mathrm{pLd}=\sigma \mathrm{c} \times \eta_{\mathrm{L}} \times 2 \mathrm{tL}$

$$
\begin{equation*}
\sigma_{\mathrm{C}}=\frac{\mathbf{p} \times \mathbf{d}}{2 \times \mathbf{t} \times \boldsymbol{\eta}_{\mathrm{L}}} \tag{1}
\end{equation*}
$$

If $\eta_{c}=$ Efficiency of circumferential joint
Efficiency $=\mathrm{A}_{\text {eff }} / \mathrm{A}$
Bursting force $=\left(\pi \mathrm{d}^{2} / 4\right) \mathrm{p}$
Resisting force $=\sigma_{L} \times A_{e f f}^{\prime}=\sigma_{L} \times \eta_{c} \times A^{\prime}=\sigma_{L} \times \eta_{c} \times \pi d t$
Where $\eta_{\mathrm{L}}=$ Efficiency of circumferential joint

Bursting force $=$ Resisting force

$$
\begin{equation*}
\sigma_{\mathrm{L}}=\frac{\mathrm{p} \times \mathbf{d}}{4 \times \mathbf{t} \times \eta_{\mathrm{C}}} \tag{2}
\end{equation*}
$$

## Sample Problem \# 1

A cylindrical tank of 750 mm internal diameter, 12 mm thickness and 1.5 m length is completely filled with an oil of specific weight $7.85 \mathrm{kN} / \mathrm{m}^{3}$ at atmospheric pressure. If the efficiency of longitudinal joints is $75 \%$ and that of circumferential joints is $45 \%$, find the pressure head of oil in the tank. Also calculate the change in volume. Take permissible tensile stress of tank plate as 120 MPa and $\mathrm{E}=200$ GPa, and $\mu=0.3$.

## SOLUTION:

Let $\mathrm{p}=$ max permissible pressure in the tank.
Then we have, $\sigma_{L}=(p \times d) /(4 \times t) \eta_{C}$

$$
\begin{aligned}
& 120=(\mathrm{p} \times 750) /(4 \times 12) 0.45 \\
& \mathrm{p}=3.456 \mathrm{MPa} .
\end{aligned}
$$

$$
\begin{gathered}
\text { Also, } \sigma_{\mathrm{C}}=(\mathrm{p} \times \mathrm{d}) /(2 \times \mathrm{t}) \eta_{\mathrm{L}} \\
120=(\mathrm{p} \times 750) /(2 \times 12) 0.75 \\
\mathrm{p}=2.88 \mathrm{MPa} .
\end{gathered}
$$

Max permissible pressure in the tank, $\mathrm{p}=2.88 \mathrm{MPa}$.

$$
\begin{aligned}
& \text { Vol. Strain, } \frac{d v}{V}=\frac{(p \times d)}{(4 \times t \times E)} \times(5-4 \times \mu) \\
& =\frac{(2.88 \times 750)}{\left(4 \times 12 \times 200 \times 10^{3}\right)} \times(5-4 \times 0.3)=8.55 \times 10^{-4} \\
& d v=8.55 \times 10^{-4} \times \mathrm{V}=8.55 \times 10^{-4} \times \frac{\pi}{4} \times 750^{2} \times 1500=0.567 \times 10^{6} \mathrm{~mm}^{3} . \\
& =0.567 \times 10^{-3} \mathrm{~m}^{3}=\underline{0.567} \text { litres. }
\end{aligned}
$$

## Sample Problem \# 2

A boiler shell is to be made of 15 mm thick plate having a limiting tensile stress of $120 \mathrm{~N} / \mathrm{mm}^{2}$. If the efficiencies of the longitudinal and circumferential joints are $70 \%$ and $30 \%$ respectively determine;
i) The maximum permissible diameter of the shell for an internal pressure of $2 \mathrm{~N} / \mathrm{mm}^{2}$.
(ii)Permissible intensity of internal pressure when the shell diameter is 1.5 m .

## SOLUTION:

(i) To find the maximum permissible diameter of the shell for an internal pressure of $2 \mathrm{~N} / \mathrm{mm}^{2}$ :
a) Let limiting Tensile Stress $=$ Circumferential stress $=\sigma_{c}=120 \mathrm{~N} / \mathrm{mm}^{2}$.

$$
\begin{array}{ll}
\text { i. e., } \quad \sigma_{c}=\frac{p \times d}{2 \times t \times \eta_{L}} \\
120= & \frac{2 \times d}{2 \times 15 \times 0.7} \\
d=1260 \mathrm{~mm}
\end{array}
$$

b) Let limiting tensile stress $=$ Longitudinal stress $=\sigma_{L}=120 \mathrm{~N} / \mathrm{mm}^{2}$.

$$
\text { i. e., } \begin{aligned}
\sigma_{L} & =\frac{p \times d}{4 \times t \times \eta_{C}} \\
120 & =\frac{2 \times d}{4 \times 15 \times 0.3} \quad . \quad d=1080 \mathrm{~mm}
\end{aligned}
$$

The maximum diameter of the cylinder in order to satisfy both the conditions $=\underline{1080} \mathrm{~mm}$.
(ii) To find the permissible pressure for an internal diameter of 1.5 m : $(\mathrm{d}=1.5 \mathrm{~m}=1500 \mathrm{~mm})$
a) Let limiting tensile stress $=$ Circumferential stress $=\sigma_{c}=$ $120 \mathrm{~N} / \mathrm{mm}^{2}$.

$$
\text { i. e., } \begin{aligned}
\sigma_{c} & =\frac{p \times d}{2 \times t \times \eta_{L}} \\
120 & =\frac{p \times 1500}{2 \times 15 \times 0.7} \\
p= & 1.68 \mathrm{~N} / \mathrm{mm}^{2} .
\end{aligned}
$$

b) Let limiting tensile stress $=$ Longitudinal stress $=\sigma_{\mathrm{L}}=120 \mathrm{~N} / \mathrm{mm}^{2}$.

$$
\begin{aligned}
& \text { i. e., } \quad \sigma_{L}=\frac{p \times d}{4 \times t \times \eta_{C}} \\
& 120=\frac{p \times 1500}{4 \times 15 \times 0.3} \\
& p=1.44 \mathrm{~N} / \mathrm{mm}^{2} .
\end{aligned}
$$

The maximum permissible pressure $=\underline{1.44} \mathrm{~N} / \mathrm{mm}^{2}$.

## Exercise Problem \# 1

A cylindrical tank of 750 mm internal diameter and 1.5 m long is to be filled with an oil of specific weight $7.85 \mathrm{kN} / \mathrm{m} 3$ under a pressure head of 365 m . If the longitudinal joint efficiency is $75 \%$ and circumferential joint efficiency is $40 \%$, find the thickness of the tank required. Also calculate the error of calculation in the quantity of oil in the tank if the volumetric strain of the tank is neglected. Take permissible tensile stress as $120 \mathrm{MPa}, \mathrm{E}=200 \mathrm{GPa}$ and $\mu=0.3$ for the tank material.
(Ans: $\mathrm{t}=12 \mathrm{~mm}$, error=0.085\%.)

## Thin Walled - Spherical Pressure Vessels

$>$ A contents are of negligible weight; a cylindrical vessel is also good with the exception of the junctions with the ends.

## Thin Walled - Spherical Pressure Vessels



Fig. 3-13 Diagrams for analysis of thin-walled cylindrical pressure vessels.

## Thin Walled - Spherical Pressure Vessels

An analogous method of analysis can be used to derive an expression for thin-walled spherical pressure vessels. By passing a section through the center of the sphere of Fig. 3-14(a), a hemisphere shown in Fig. 314(b) is isolated. By using the same notation as before, an equation identical to Eq. 3-25 can be derived. However, for a sphere, any section that passes through the center of the sphere yields the same result whatever the inclination of the element's side; see Fig. 3-14(c). Hence, the maximum membrane stresses for thin-walled spherical pressure vessels are


## Thin Walled - Spherical Pressure Vessels

## EXAMPLE 3-4

Consider a steel spherical pressure vessel of radius 1000 mm having a wall thickness of 10 mm . (a) Determine the maximum membrane stresses caused by an internal pressure of 0.80 MPa . (b) Calculate the change in diameter in the sphere caused by pressurization. Let $E=200 \mathrm{GPa}$, and $\nu=0.25$. Assume that $r_{i} \approx r_{o}$ $\approx r$.

## Solution

The maximum membrane normal stresses follow directly from Eq. 3-26.

$$
\sigma_{1}=\sigma_{2}=\frac{p r}{2 t}=\frac{0.80 \times 1}{2 \times 10 \times 10^{-3}}=40 \mathrm{MPa}
$$

The same procedure as in the previous example can be used for finding the expansion of the sphere due to pressurization. Hence, if $\Delta$ is the increase in the radius $r$ due to this cause, $\Delta=\varepsilon_{1} r$, where $\varepsilon_{1}$ is the membrane strain on the great circle. However, from the first expression in Eq. 3-14, one has

$$
\varepsilon_{1}=\frac{\sigma_{1}}{E}-\nu \frac{\sigma_{2}}{E}=\frac{40}{200 \times 10^{3}}-\frac{40}{4 \times 200 \times 10^{3}}=0.15 \times 10^{-3} \mathrm{~mm} / \mathrm{mm}
$$

$$
\text { Hence, } \quad \Delta=\varepsilon_{1} r=0.15 \times 10^{-3} \times 10^{3}=0.15 \mathrm{~mm}
$$

## Thin Walled - Spherical Pressure Vessels

It is instructive to note that for comparable size and wall thickness, the maximum normal stress in a spherical pressure vessel is only about onehalf as large as that in a cylindrical one. The reason for this can be clarified by making reference to Figs. 3-17 and 3-18. In a cylindrical pressure vessel, the longitudinal stresses, $\sigma_{2}$, parallel to the vessel's axis, do not contribute to maintaining the equilibrium of the internal pressure $p$ acting on the curved surface; whereas in a spherical vessel, a system of equal stresses resists the applied internal pressure. These stresses, given by Eqs. 3-24-3-26, are treated as biaxial, although the internal pressure $p$ acting on the wall causes local compressive stresses on the inside equal


Fig. 3-17 An element of a


Fig. 3-18 An element of a

## Thin Walled - Spherical Pressure Vessels

to this pressure. As already pointed out in Example 3-3, such stresses are small in comparison with the membrane stresses $\sigma_{1}$ and $\sigma_{2}$, and are generally ignored for thin-walled pressure vessels. A more complete discussion of this problem is given in Section 3-13 and Example 3-6. A much more important problem arises at geometrical changes in the shape of a vessel. These can cause a disturbance in the membrane action. An illustration of this condition is given in Fig. 3-19 using the numerical results found in Examples 3-3 and 3-4.


Fig. 3-17 An element of a thin-walled cylindrical pressure vessel.


Fig. 3-18 An element of a thin-walled spherical pressure vessel.

If a cylindrical pressure vessel has hemispherical ends, as shown in

## Thin

Fig. 3-19(a), and if initially the cylinder and the heads were independent of each other, under pressurization they would tend to expand, as shown by the dashed lines. In general, the cylinder and the ends would expand by different amounts and would tend to create a discontinuity in the wall, as shown at $A$. However, physical continuity of the wall must be maintained by local bending and shear stresses in the neighborhood of the juncture, as shown in Fig. 3-19(b). If instead of relatively flexible hemi-

(a)


Detail A
(b)

(c) Deformation of the same cylindrical pressure vessel at a flat head

Fig. 3-19 Exaggerated deformations of pressure vessels at discontinuities.

## Thin Walled - Spherical Pressure Vessels

spherical ends, thick end plates are used, the local bending and shear stresses increase considerably; see Fig. 3-19(c). For this reason, the ends (heads) of pressure vessels must be very carefully designed. ${ }^{9}$ Flat ends are very undesirable.

A majority of pressure vessels are manufactured from curved sheets that are joined together by means of welding. Examples of welds used in pressure vessels are shown in Fig. 3-20, with preference given to the different types of butt joints. Some additional comments on welded joints may be found in Section 1-14.

In conclusion, it must be emphasized that the formulas derived for thinwalled pressure vessels in the preceding section should be used only for cases of internal pressure. If a vessel is to be designed for external pressure, as in the case of a vacuum tank or a submarine, instability (buckling) of the walls may occur, and stress calculations based on the previous formulas can be meaningless.


Fig. 3-20 Examples of welds used in pressure vessels. (a) Double-fillet lap joint, and (b) double-welded butt joint with V-grooves.

## Software Images of Stresses on Critical Joints



## Abaqus Software Images of Stresses on Cylinder



S, Mises
S, Mises
(Avg: 75\%)
(Avg: 75\%)






## Thank-you for Listening!

