

Thin Walled Cylinders

Mechanics of Solids-2

Lecture # 1

Engr. Shad Muhammad
Lecturer
Department of Civil Engineering
COMSATS University Islamabad, Sahiwal Campus.



Table of Content

2

- Thin and Thick Walled Cylinders
- Applications of Thin and Thick Walled Cylinders
- Three Different forms of Stresses in Cylinders
- Derivation of Formulae for Stresses in Thin Walled Cylinders
- Application of Stresses on Real World Engineering Problems.

Thin And Thick Cylinders- Introduction

3

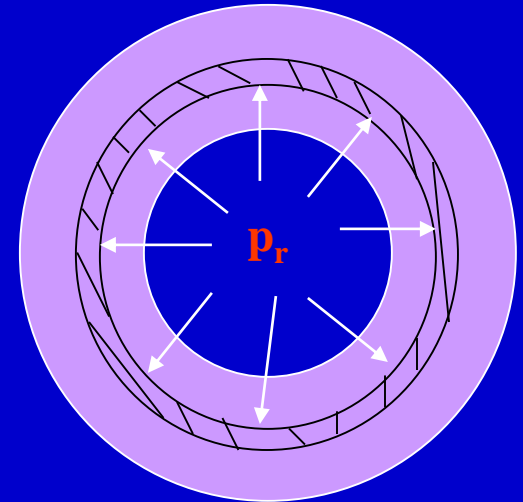
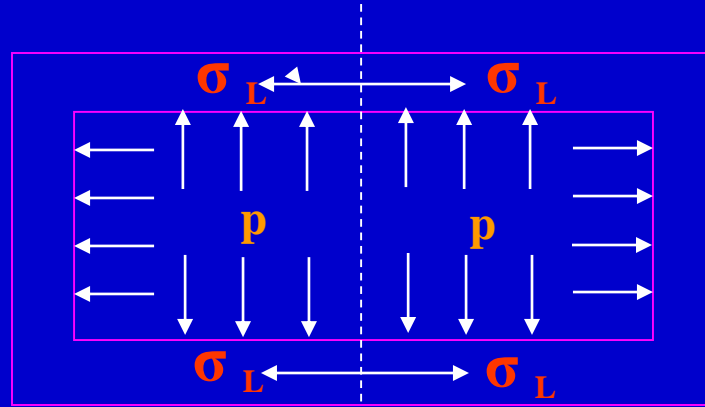
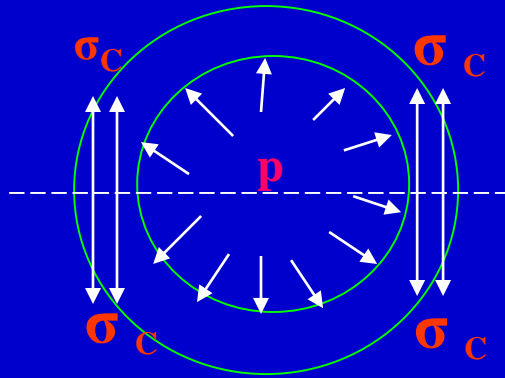
- In many engineering applications, cylinders are frequently used for transporting or storing of liquids, gases or fluids.

Eg: Pipes, Boilers, storage tanks etc.

- These cylinders are subjected to **fluid pressures**. When a cylinder is subjected to a internal pressure, at any point on the cylinder wall, **three types of stresses** are induced on **three mutually perpendicular planes**.

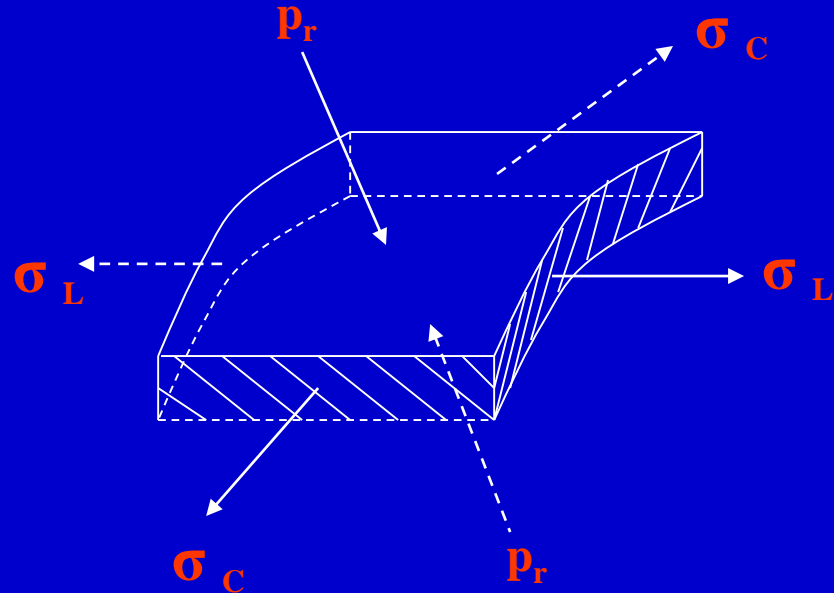
They are,

1. Hoop or Circumferential Stress (σ_C) – This is directed along the tangent to the circumference and tensile in nature. Thus, there will be increase in diameter.
2. Longitudinal Stress (σ_L) – This stress is directed along the length of the cylinder. This is also tensile in nature and tends to increase the length.
3. Radial pressure (p_r) – It is compressive in nature. Its magnitude is equal to fluid pressure on the inside wall and zero on the outer wall if it is open to atmosphere.



1. Hoop Stress (σ_c) 2. Longitudinal Stress (σ_L) 3. Radial Stress (p_r)

Element on the cylinder wall subjected to these three stresses



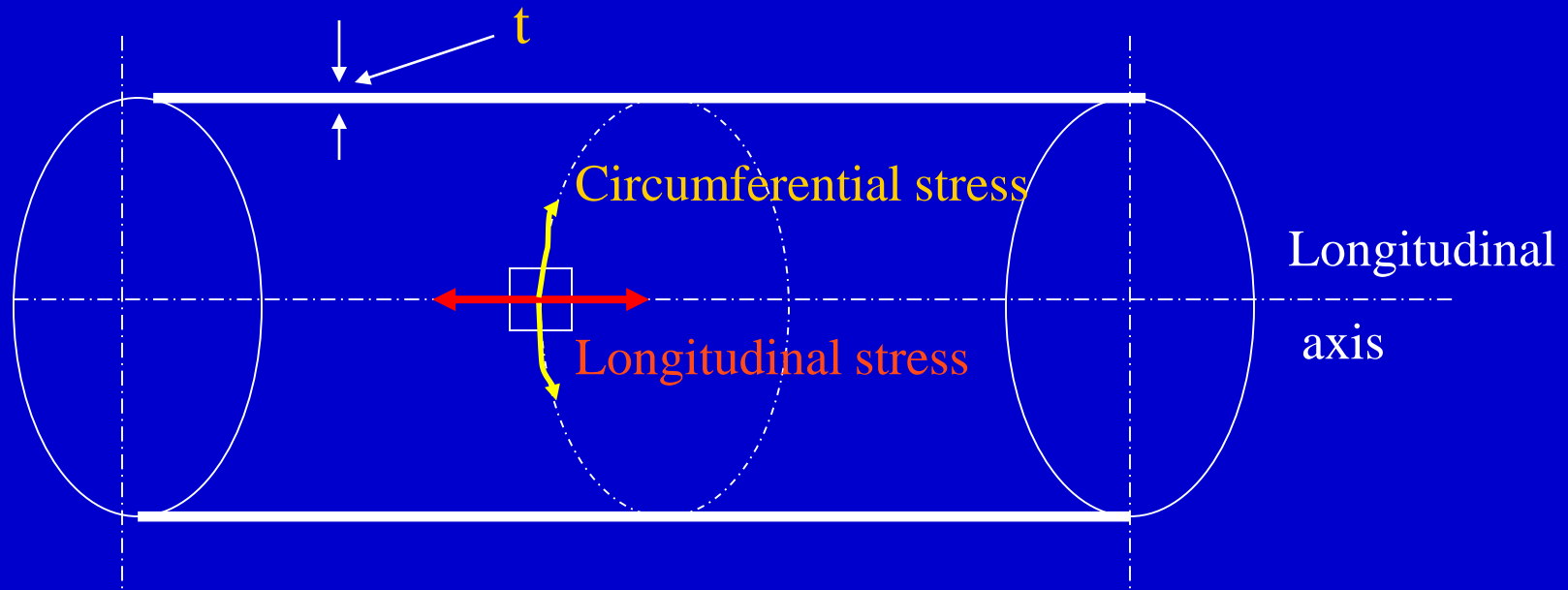
THIN WALLED CYLINDERS

INTRODUCTION:

A cylinder or spherical shell is considered to be thin when the metal thickness is small compared to internal diameter.

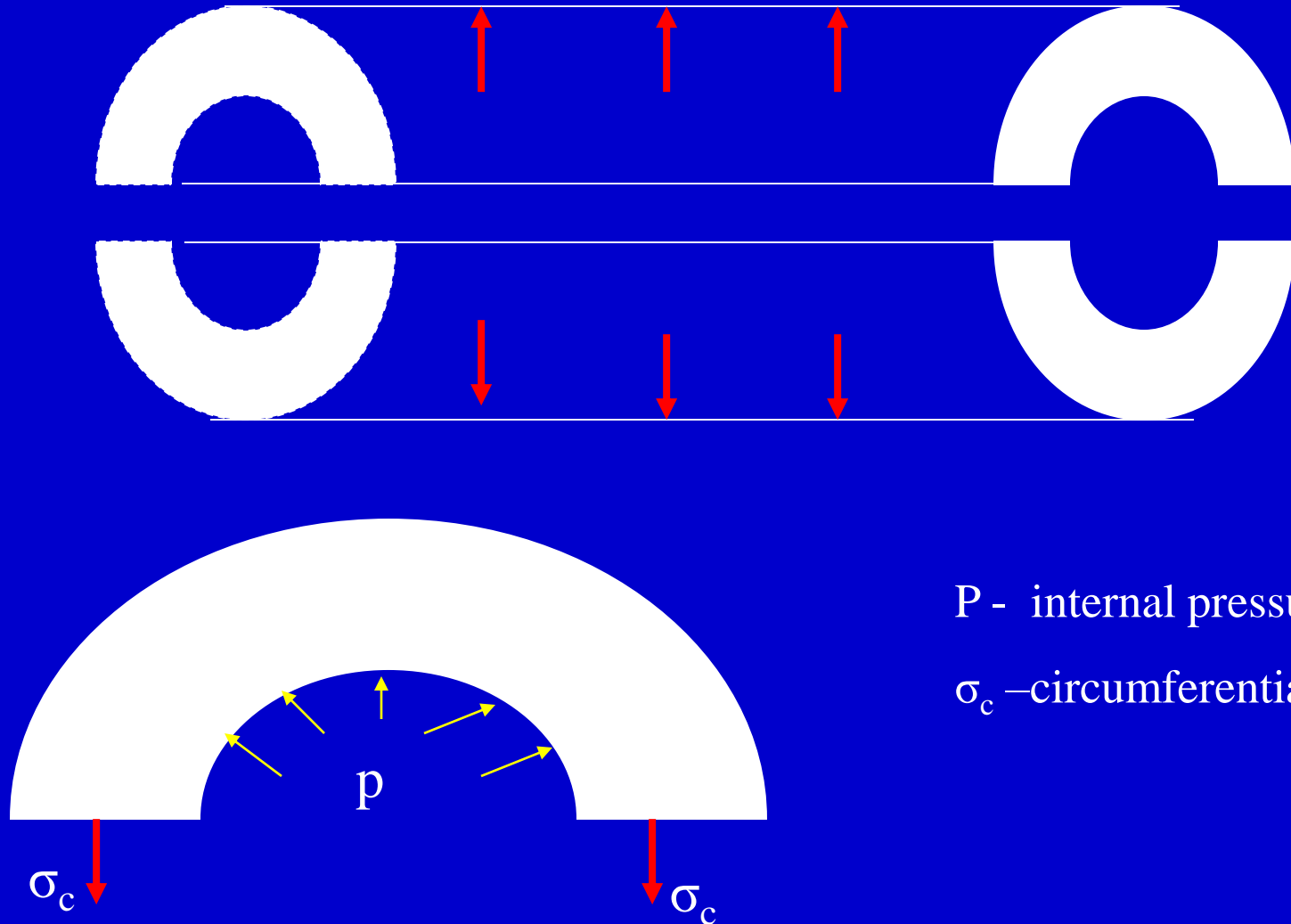
i. e., when the wall thickness, 't' is equal to or less than ' $d/20$ ', where 'd' is the internal diameter of the cylinder or shell, we consider the cylinder or shell to be thin, otherwise thick.

Magnitude of radial pressure is very small compared to other two stresses in case of thin cylinders and hence neglected.



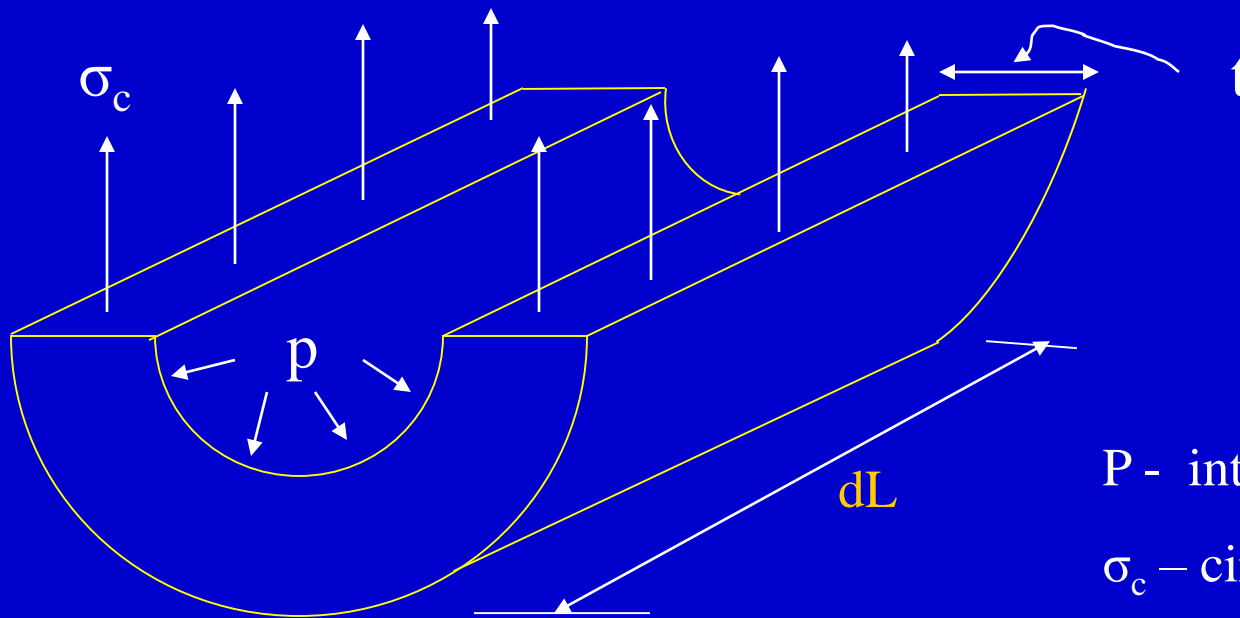
The stress acting along the circumference of the cylinder is called circumferential stresses whereas the stress acting along the length of the cylinder (i.e., in the longitudinal direction) is known as longitudinal stresses.

The bursting will take place if the force due to internal (fluid) pressure (acting vertically upwards and downwards) is more than the resisting force due to circumferential stress set up in the material.



P - internal pressure (stress)

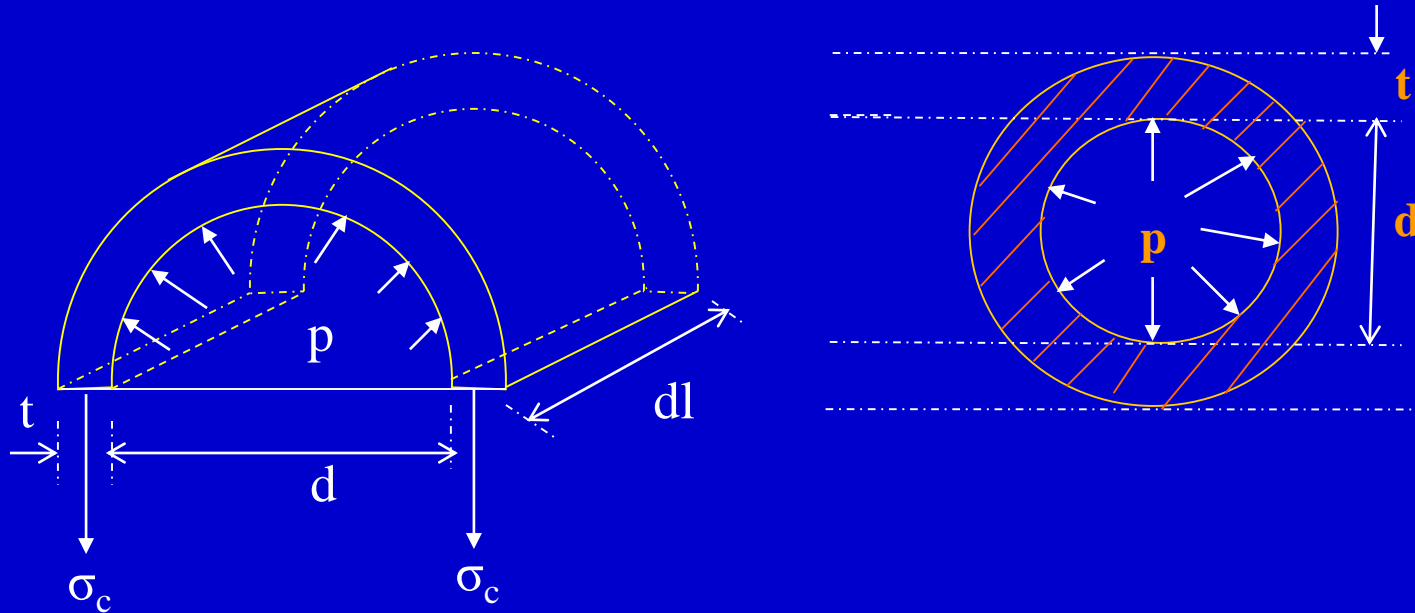
σ_c - circumferential stress



P - internal pressure (stress)

σ_c - circumferential stress

EVALUATION OF CIRCUMFERENTIAL or HOOP STRESS (σ_c):



Consider a thin cylinder closed at both ends and subjected to internal pressure 'p' as shown in the figure.

Let d = Internal diameter, t = Thickness of the wall

L = Length of the cylinder.

To determine the Bursting force across the diameter:

Consider a small length ' dl ' of the cylinder and an elementary area ' dA ' as shown in the figure.

Force on the elementary area,

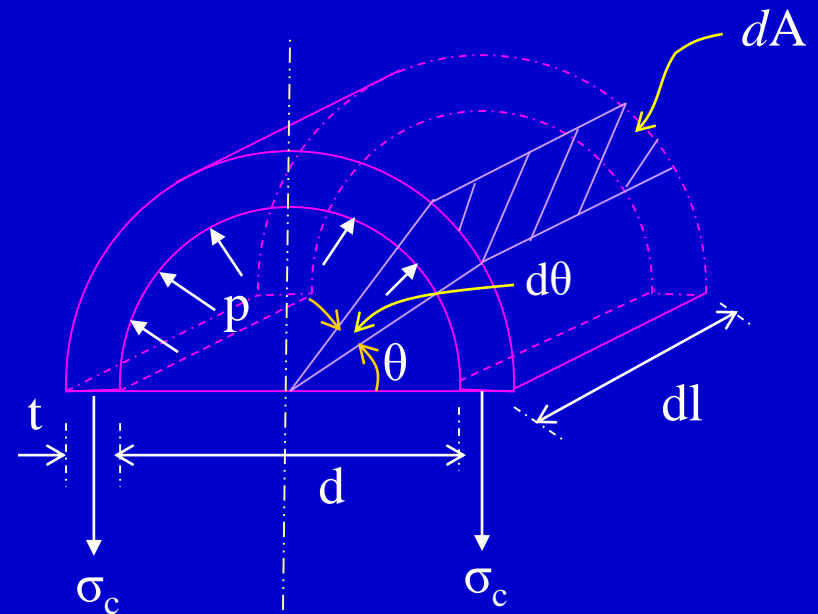
$$\begin{aligned} dF &= p \times dA = p \times r \times dl \times d\theta \\ &= p \times \frac{d}{2} \times dl \times d\theta \end{aligned}$$

Horizontal component of this force

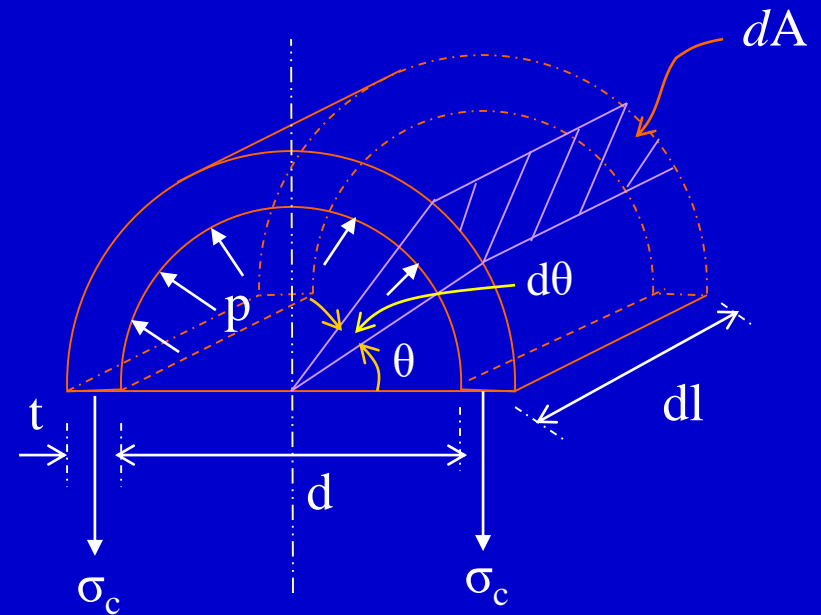
$$dF_x = p \times \frac{d}{2} \times dl \times \cos \theta \times d\theta$$

Vertical component of this force

$$dF_y = p \times \frac{d}{2} \times dl \times \sin \theta \times d\theta$$



The horizontal components cancel out when integrated over semi-circular portion as there will be another equal and opposite horizontal component on the other side of the vertical axis.



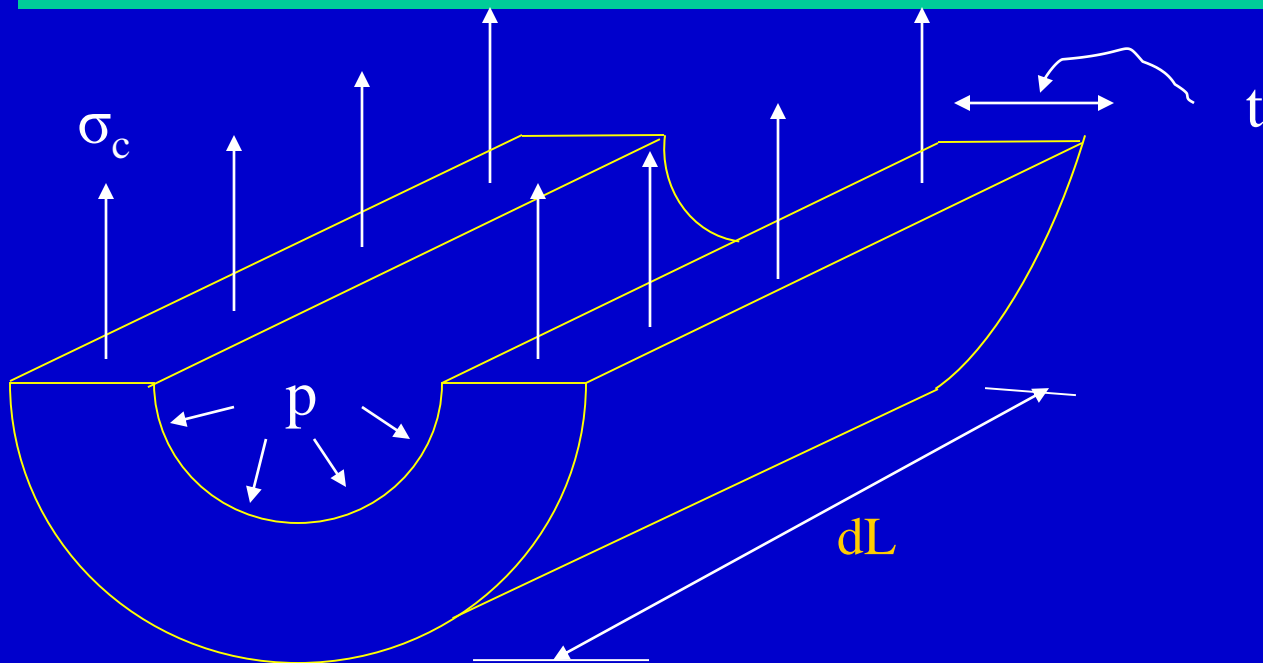
$$\begin{aligned}
 \therefore \text{Total diametrical bursting force} &= \int_0^{\pi} p \times \frac{d}{2} \times dl \times \sin \theta \times d\theta \\
 &= p \times \frac{d}{2} \times dl \times [-\cos \theta]_0^{\pi} = \underline{p \times d \times dl} \\
 &= p \times \text{projected area of the curved surface.}
 \end{aligned}$$

\therefore Resisting force (due to circumferential stress σ_c) = $2 \times \sigma_c \times t \times dl$

Under equilibrium, Resisting force = Bursting force

$$\text{i.e., } 2 \times \sigma_c \times t \times dl = p \times d \times dl$$

$$\therefore \text{Circumferential stress, } \sigma_c = \frac{p \times d}{2 \times t} \dots \dots \dots (1)$$



Assumed as rectangular



Force due to fluid pressure = $p \times \text{area on which } p \text{ is acting} = p \times (d \times L)$
(bursting force)

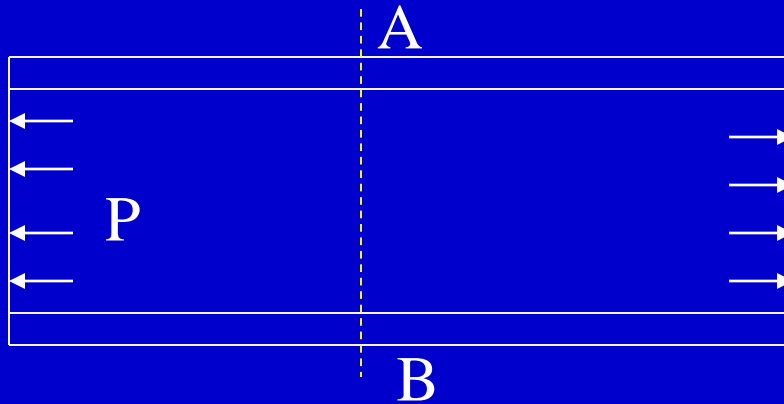
Force due to circumferential stress = $\sigma_c \times \text{area on which } \sigma_c \text{ is acting}$
(resisting force) = $\sigma_c \times (L \times t + L \times t) = \sigma_c \times 2 L \times t$

Under equilibrium bursting force = resisting force

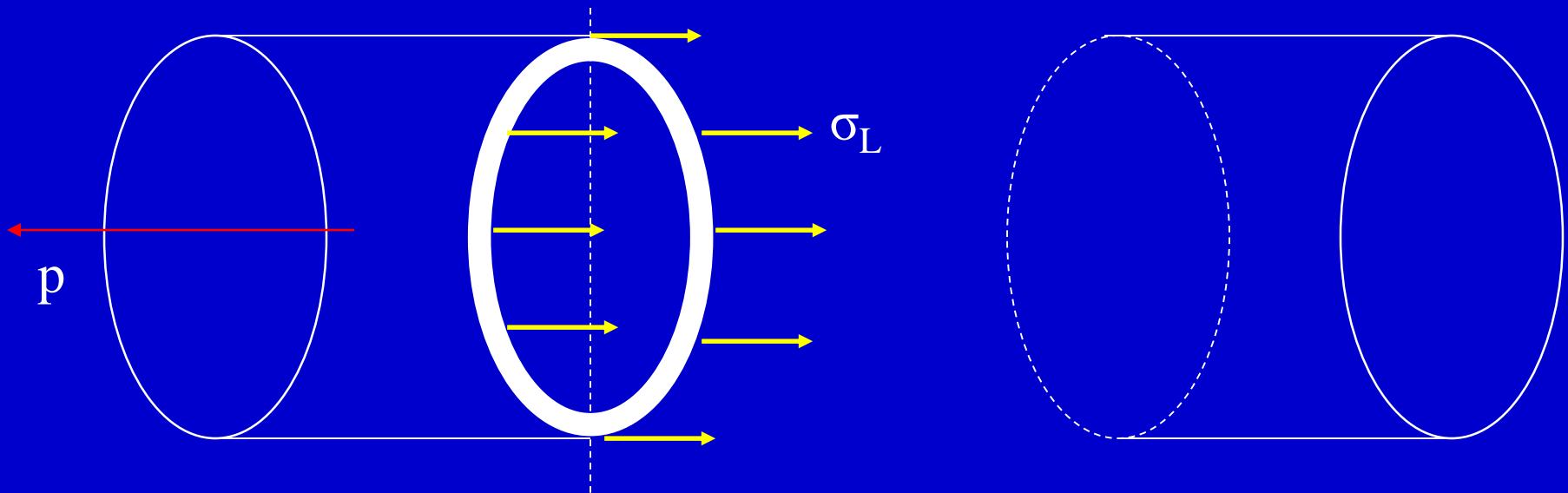
$$p \times (d \times L) = \sigma_c \times 2 L \times t$$

$$\therefore \text{Circumferential stress, } \sigma_c = \frac{p \times d}{2 \times t} \dots\dots\dots (1)$$

LONGITUDINAL STRESS (σ_L):

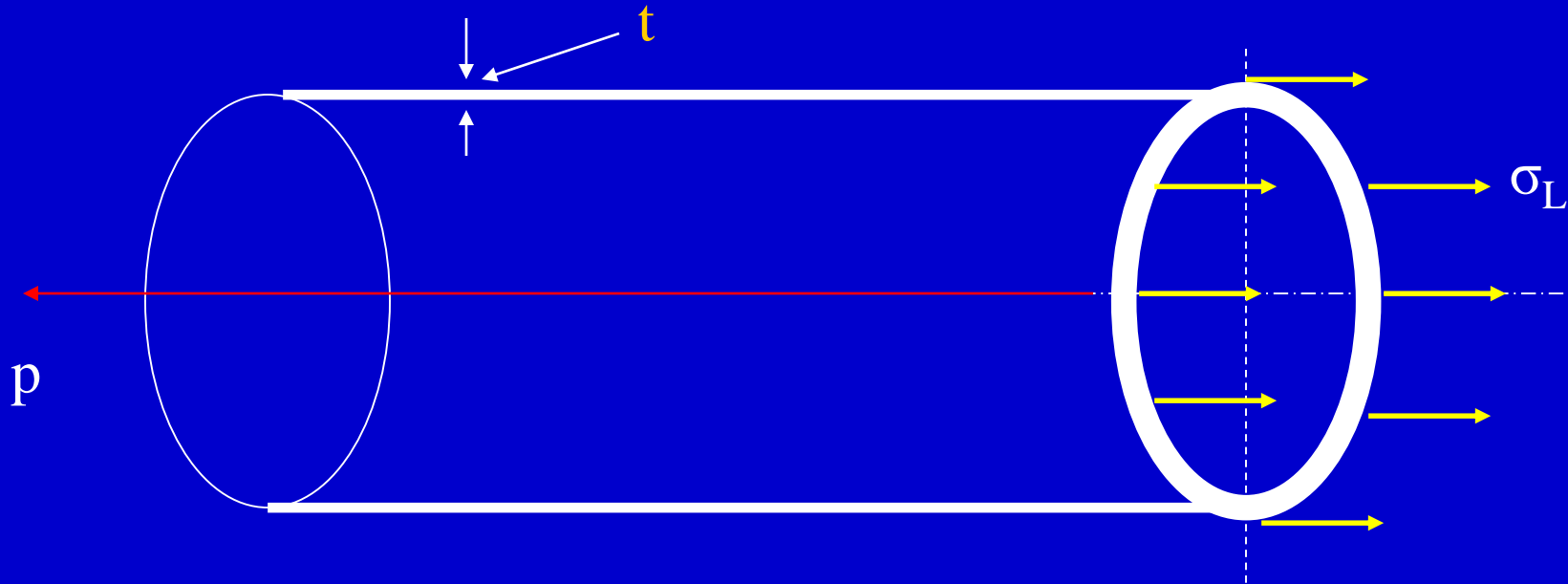


The bursting of the cylinder takes place along the section AB



The force, due to pressure of the fluid, acting at the ends of the thin cylinder, tends to burst the cylinder as shown in figure

EVALUATION OF LONGITUDINAL STRESS (σ_L):



$$\text{Longitudinal bursting force (on the end of cylinder)} = p \times \frac{\pi}{4} \times d^2$$

$$\text{Area of cross section resisting this force} = \pi \times d \times t$$

Let σ_L = Longitudinal stress of the material of the cylinder.

$$\therefore \text{Resisting force} = \sigma_L \times \pi \times d \times t$$

Under equilibrium, bursting force = resisting force

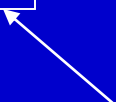
$$\text{i.e., } p \times \frac{\pi}{4} \times d^2 = \sigma_L \times \pi \times d \times t$$

$$\therefore \text{ Longitudinal stress, } \sigma_L = \frac{p \times d}{4 \times t} \dots\dots\dots (2)$$

$$\text{From eqs (1) \& (2), } \underline{\sigma_C = 2 \times \sigma_L}$$

$$\begin{aligned}\text{Force due to fluid pressure} &= p \times \text{area on which } p \text{ is acting} \\ &= p \times \frac{\pi}{4} \times d^2\end{aligned}$$

$$\begin{aligned}\text{Resisting force} &= \sigma_L \times \text{area on which } \sigma_L \text{ is acting} \\ &= \sigma_L \times \boxed{\pi \times d} \times t\end{aligned}$$

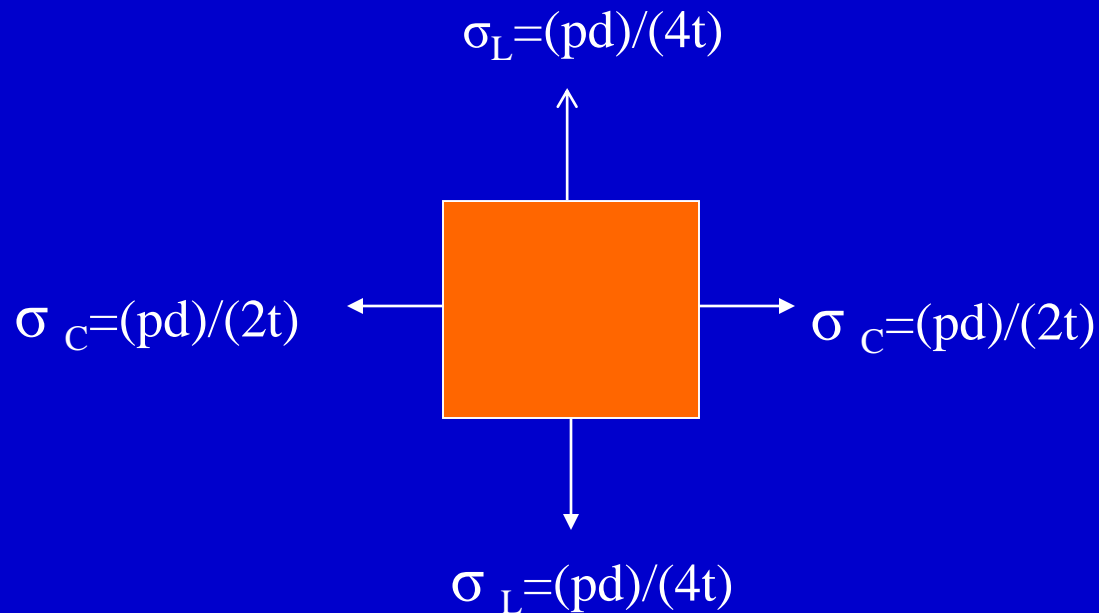
 circumference

Under equilibrium, bursting force = resisting force

$$\text{i.e., } p \times \frac{\pi}{4} \times d^2 = \sigma_L \times \pi \times d \times t$$

$$\therefore \text{ Longitudinal stress, } \sigma_L = \frac{p \times d}{4 \times t} \dots\dots\dots (2)$$

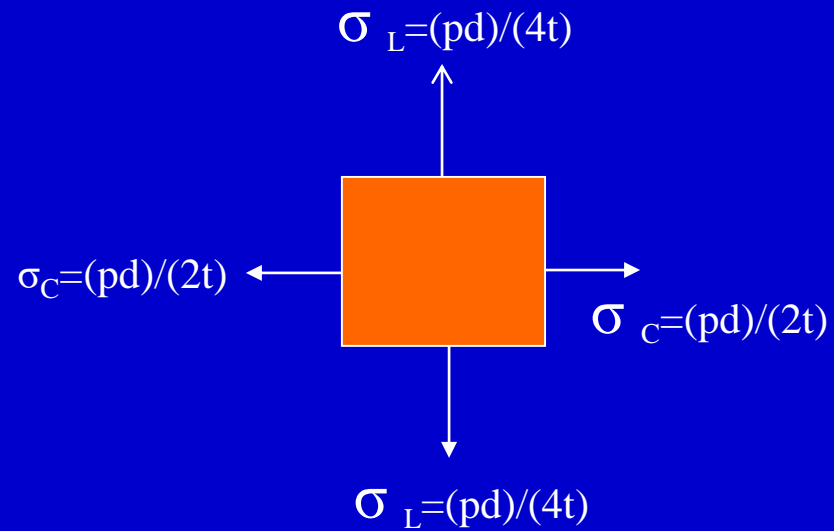
EVALUATION OF STRAINS



A point on the surface of thin cylinder is subjected to biaxial stress system, (Hoop stress and Longitudinal stress) mutually perpendicular to each other, as shown in the figure. The strains due to these stresses i.e., circumferential and longitudinal are obtained by applying Hooke's law and Poisson's theory for elastic materials.

Circumferential strain, ϵ_C :

$$\begin{aligned}\epsilon_C &= \frac{\sigma_C}{E} - \mu \times \frac{\sigma_L}{E} \\ &= 2 \times \frac{\sigma_L}{E} - \mu \times \frac{\sigma_L}{E} \\ &= \frac{\sigma_L}{E} \times (2 - \mu)\end{aligned}$$



$$\text{i.e., } \underline{\epsilon_C = \frac{\delta d}{d} = \frac{p \times d}{4 \times t \times E} \times (2 - \mu) \dots \dots \dots (3)}$$

Longitudinal strain, ϵ_L :

$$\begin{aligned}\epsilon_L &= \frac{\sigma_L}{E} - \mu \times \frac{\sigma_C}{E} \\ &= \frac{\sigma_L}{E} - \mu \times \frac{(2 \times \sigma_L)}{E} \\ &= \frac{\sigma_L}{E} \times (1 - 2 \times \mu)\end{aligned}$$

$$\text{i.e., } \epsilon_L = \frac{\delta l}{L} = \frac{p \times d}{4 \times t \times E} \times (1 - 2 \times \mu) \dots \dots \dots (4)$$

VOLUMETRIC STRAIN, $\frac{\delta V}{V}$

Change in volume = δV = final volume – original volume

original volume = V = area of cylindrical shell \times length

$$= \frac{\pi d^2}{4} L$$

final volume = final area of cross section \times final length

$$\begin{aligned} &= \frac{\pi}{4} [d + \delta d]^2 \times [L + \delta L] \\ &= \frac{\pi}{4} [d^2 + (\delta d)^2 + 2d\delta d] \times [L + \delta L] \\ &= \frac{\pi}{4} [d^2 L + (\delta d)^2 L + 2Ld\delta d + d^2 \delta L + (\delta d)^2 \delta L + 2d\delta d \delta L] \end{aligned}$$

neglecting the smaller quantities such as $(\delta d)^2 L$, $(\delta d)^2 \delta L$ and $2d\delta d \delta L$

$$\text{Final volume} = \frac{\pi}{4} [d^2 L + 2Ld\delta d + d^2 \delta L]$$

$$\text{change in volume } \delta V = \frac{\pi}{4} [d^2 L + 2Ld\delta d + d^2 \delta L] - \frac{\pi}{4} [d]^2 L$$

$$\delta V = \frac{\pi}{4} [2Ld\delta d + d^2 \delta L]$$

$$\frac{dv}{V} = \frac{\frac{\pi}{4} [2dL\delta d + \delta Ld^2]}{\frac{\pi}{4} \times d^2 \times L}$$

$$= \frac{\delta L}{L} + 2 \times \frac{\delta d}{d}$$

$$\frac{dV}{V} = \epsilon_L + 2 \times \epsilon_C$$

$$= \frac{p \times d}{4 \times t \times E} (1 - 2 \times \mu) + 2 \times \frac{p \times d}{4 \times t \times E} (2 - \mu)$$

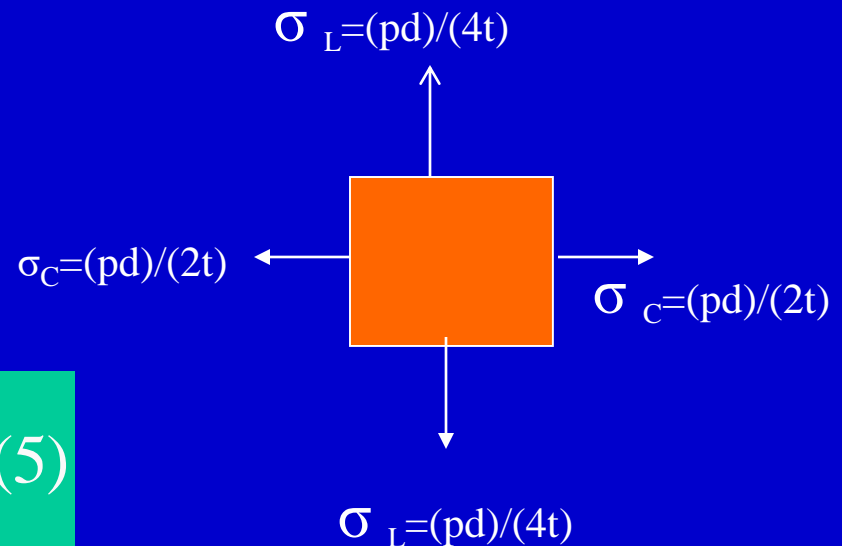
i.e.,
$$\underline{\underline{\frac{dv}{V} = \frac{p \times d}{4 \times t \times E} (5 - 4 \times \mu) \dots \dots \dots (5)}}$$

Maximum Shear Stress:

There are two principal stresses at any point, viz., circumferential and longitudinal. Both these stresses are normal and act perpendicular to each other.

$$\therefore \text{Maximum Shear stress, } \tau_{\max} = \frac{\sigma_C - \sigma_L}{2}$$
$$= \frac{\frac{pd}{2t} - \frac{pd}{4t}}{2}$$

i.e., $\tau_{\max} = \frac{pd}{8t} \dots \dots \dots (5)$



ILLUSTRATIVE PROBLEMS

PROBLEM 1:

A thin cylindrical shell is 3m long and 1m in internal diameter. It is subjected to internal pressure of 1.2 MPa. If the thickness of the sheet is 12mm, find the circumferential stress, longitudinal stress, changes in diameter, length and volume . Take $E=200$ GPa and $\mu=0.3$.

SOLUTION:

1. Circumferential stress, σ_C :

$$\begin{aligned}\sigma_C &= (p \times d) / (2 \times t) \\ &= (1.2 \times 1000) / (2 \times 12) \\ &= \underline{50 \text{ N/mm}^2} = \underline{50 \text{ MPa}} \text{ (Tensile)}.\end{aligned}$$

2. Longitudinal stress, σ_L :

$$\begin{aligned}\sigma_L &= (p \times d) / (4 \times t) \\ &= \sigma_C / 2 = 50 / 2 \\ &= \underline{25 \text{ N/mm}^2} = \underline{25 \text{ MPa}} \text{ (Tensile)}.\end{aligned}$$

3. Circumferential strain, ϵ_c :

$$\begin{aligned}\epsilon_c &= \frac{(p \times d)}{(4 \times t)} \times \frac{(2 - \mu)}{E} \\ &= \frac{(1.2 \times 1000)}{(4 \times 12)} \times \frac{(2 - 0.3)}{200 \times 10^3} \\ &= \underline{2.125 \times 10^{-04}} \text{ (Increase)}\end{aligned}$$

$$\begin{aligned}\text{Change in diameter, } \delta d &= \epsilon_c \times d \\ &= 2.125 \times 10^{-04} \times 1000 = \underline{0.2125} \text{ mm (Increase).}\end{aligned}$$

4. Longitudinal strain, ϵ_L :

$$\begin{aligned}\epsilon_L &= \frac{(p \times d)}{(4 \times t)} \times \frac{(1 - 2 \times \mu)}{E} \\ &= \frac{(1.2 \times 1000)}{(4 \times 12)} \times \frac{(1 - 2 \times 0.3)}{200 \times 10^3} \\ &= \underline{5 \times 10^{-05}} \text{ (Increase)}\end{aligned}$$

$$\text{Change in length} = \epsilon_L \times L = 5 \times 10^{-05} \times 3000 = \underline{0.15} \text{ mm (Increase).}$$

Volumetric strain, $\frac{dv}{V}$:

$$\frac{dv}{V} = \frac{(p \times d)}{(4 \times t) \times E} \times (5 - 4 \times \mu)$$

$$= \frac{(1.2 \times 1000)}{(4 \times 12) \times 200 \times 10^3} \times (5 - 4 \times 0.3)$$

$$= 4.75 \times 10^{-4} \text{ (Increase)}$$

$$\therefore \text{Change in volume, } dv = 4.75 \times 10^{-4} \times V$$

$$= 4.75 \times 10^{-4} \times \frac{\pi}{4} \times 1000^2 \times 3000$$

$$= 1.11919 \times 10^6 \text{ mm}^3 = 1.11919 \times 10^{-3} \text{ m}^3$$

$$= \underline{1.11919 \text{ Litres}}.$$

PROBLEM 2:

A copper tube having 45mm internal diameter and 1.5mm wall thickness is closed at its ends by plugs which are at 450mm apart. The tube is subjected to internal pressure of 3 MPa and at the same time pulled in axial direction with a force of 3 kN. Compute: i) the change in length between the plugs ii) the change in internal diameter of the tube. Take $E_{CU} = 100 \text{ GPa}$, and $\mu_{CU} = 0.3$.

SOLUTION:

A] Due to Fluid pressure of 3 MPa:

$$\begin{aligned}\text{Longitudinal stress, } \sigma_L &= (p \times d) / (4 \times t) \\ &= (3 \times 45) / (4 \times 1.5) = 22.50 \text{ N/mm}^2 = 22.50 \text{ MPa.}\end{aligned}$$

$$\text{Long. strain, } \varepsilon_L = \frac{(p \times d)}{4 \times t} \times \frac{(1 - 2 \times \mu)}{E}$$

$$= \frac{22.5 \times (1 - 2 \times 0.3)}{100 \times 10^3} = \underline{9 \times 10^{-5}}$$

$$\text{Change in length, } \delta_L = \varepsilon_L \times L = 9 \times 10^{-5} \times 450 = \underline{+0.0405 \text{ mm}} \text{ (increase)}$$

$$Pd/4t = 22.5$$

$$\text{Circumferential strain } \epsilon_c = \frac{(p \times d)}{(4 \times t)} \times \frac{(2 - \mu)}{E}$$

$$= \frac{22.5 \times (2 - 0.3)}{100 \times 10^3} = \underline{3.825 \times 10^{-4}}$$

$$\begin{aligned} \text{Change in diameter, } \delta_d &= \epsilon_c \times d = 3.825 \times 10^{-4} \times 45 \\ &= + \underline{0.0172 \text{ mm}} \text{ (increase)} \end{aligned}$$

B] Due to Pull of 3 kN (P=3kN):

$$\begin{aligned} \text{Area of cross section of copper tube, } A_c &= \pi \times d \times t \\ &= \pi \times 45 \times 1.5 = 212.06 \text{ mm}^2 \end{aligned}$$

$$\begin{aligned} \text{Longitudinal strain, } \epsilon_L &= \text{direct stress}/E = \sigma/E = P/(A_c \times E) \\ &= 3 \times 10^3 / (212.06 \times 100 \times 10^3) \\ &= \underline{1.415 \times 10^{-4}} \end{aligned}$$

$$\text{Change in length, } \delta_L = \epsilon_L \times L = 1.415 \times 10^{-4} \times 450 = \underline{+0.0637 \text{ mm}} \text{ (increase)}$$

Lateral strain, $\epsilon_{\text{lat}} = -\mu \times \text{Longitudinal strain} = -\mu \times \epsilon_L$
 $= -0.3 \times 1.415 \times 10^{-4} = -4.245 \times 10^{-5}$

Change in diameter, $\delta_d = \epsilon_{\text{lat}} \times d = -4.245 \times 10^{-5} \times 45$
 $= \underline{-1.91 \times 10^{-3} \text{ mm}} \text{ (decrease)}$

C) Changes due to combined effects:

Change in length $= 0.0405 + 0.0637 = \underline{+0.1042 \text{ mm}}$ (increase)

Change in diameter $= 0.01721 - 1.91 \times 10^{-3} = \underline{+0.0153 \text{ mm}}$ (increase)

PROBLEM 3:

A cylindrical boiler is 800mm in diameter and 1m length. It is required to withstand a pressure of 100m of water. If the permissible tensile stress is 20N/mm², permissible shear stress is 8N/mm² and permissible change in diameter is 0.2mm, find the minimum thickness of the metal required. Take $E = 200\text{GPa}$, and $\mu = 0.3$.

SOLUTION:

$$\begin{aligned}\text{Fluid pressure, } p &= 100\text{m of water} = 100 \times 9.81 \times 10^3 \text{ N/m}^2 \\ &= 0.981 \text{ N/mm}^2.\end{aligned}$$

1. Thickness from Hoop Stress consideration: (Hoop stress is critical than long. Stress)

$$\sigma_c = (p \times d) / (2 \times t)$$

$$20 = (0.981 \times 800) / (2 \times t)$$

$$t = \underline{19.62} \text{ mm}$$

2. Thickness from Shear Stress consideration:

$$\tau_{\max} = \frac{(p \times d)}{(8 \times t)}$$

$$8 = \frac{(0.981 \times 800)}{(8 \times t)}$$

$$\therefore t = \underline{12.26\text{mm.}}$$

3. Thickness from permissible change in diameter consideration

($\delta d = 0.2\text{mm}$):

$$\frac{\delta d}{d} = \frac{(p \times d)}{(4 \times t)} \times \frac{(2 - \mu)}{E}$$

$$\frac{0.2}{800} = \frac{(0.981 \times 800)}{(4 \times t)} \times \frac{(2 - 0.3)}{200 \times 10^3}$$

$$t = \underline{6.67\text{mm}}$$

Therefore, required thickness, $t = \underline{19.62 \text{ mm.}}$

PROBLEM 4:

A cylindrical boiler has 450mm in internal diameter, 12mm thick and 0.9m long. It is initially filled with water at atmospheric pressure. Determine the pressure at which an additional water of 0.187 liters may be pumped into the cylinder by considering water to be incompressible. Take $E = 200 \text{ GPa}$, and $\mu = 0.3$.

SOLUTION:

Additional volume of water, $\delta V = 0.187 \text{ liters} = 0.187 \times 10^{-3} \text{ m}^3$
 $= 187 \times 10^3 \text{ mm}^3$

$$V = \frac{\pi}{4} \times 450^2 \times (0.9 \times 10^3) = 143.14 \times 10^6 \text{ mm}^3$$

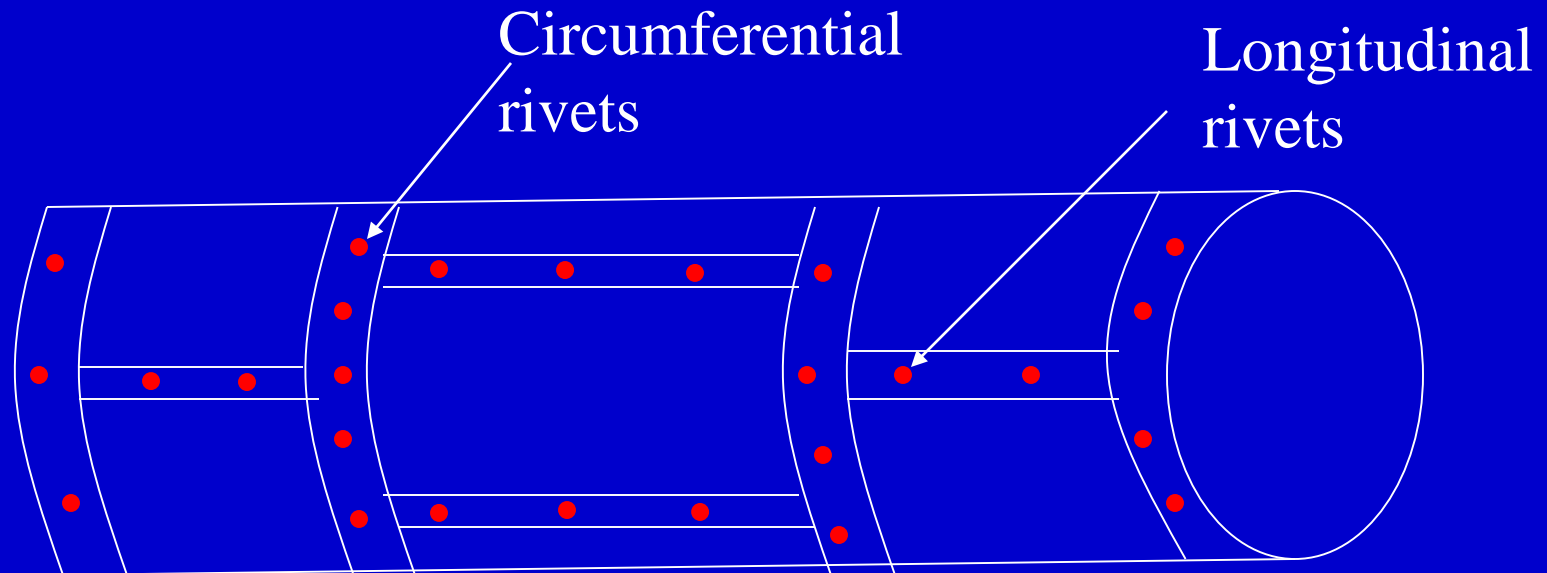
$$\frac{dV}{V} = \frac{p \times d}{4 \times t \times E} (5 - 4 \times \mu)$$

$$\frac{187 \times 10^3}{143.14 \times 10^6} = \frac{p \times 450}{4 \times 12 \times 200 \times 10^3} (5 - 4 \times 0.33)$$

Solving, $p = 7.33 \text{ N/mm}^2$

JOINT EFFICIENCY

Steel plates of only particular lengths and width are available. Hence whenever larger size cylinders (like boilers) are required, a number of plates are to be connected. This is achieved by using riveting in circumferential and longitudinal directions as shown in figure. Due to the holes for rivets, the net area of cross section decreases and hence the stresses increase.



JOINT EFFICIENCY

The cylindrical shells like boilers are having two types of joints namely Longitudinal and Circumferential joints. Due to the holes for rivets, the net area of cross section decreases and hence the stresses increase. If the efficiencies of these joints are known, the stresses can be calculated as follows.

Let η_L = Efficiency of Longitudinal joint
and η_C = Efficiency of Circumferential joint.

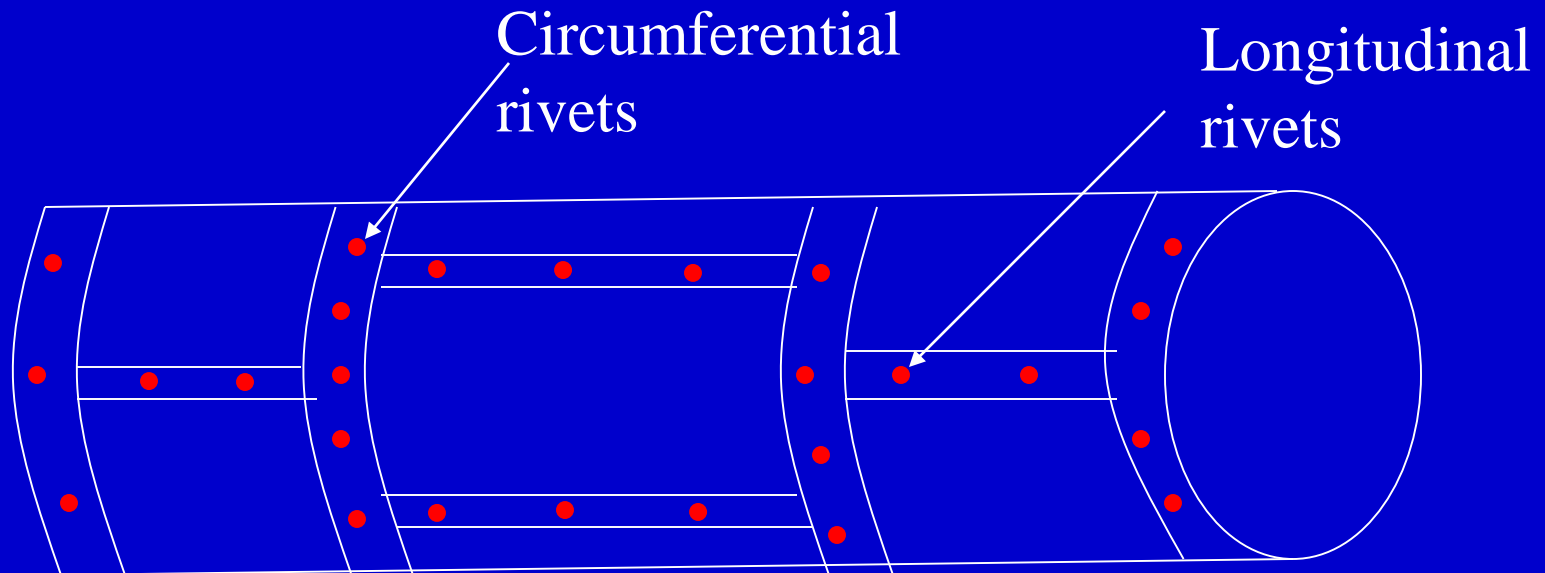
Circumferential stress is given by,

$$\sigma_C = \frac{p \times d}{2 \times t \times \eta_L} \dots\dots\dots(1)$$

Longitudinal stress is given by,

$$\sigma_L = \frac{p \times d}{4 \times t \times \eta_c} \dots\dots\dots(2)$$

Note: In longitudinal joint, the circumferential stress is developed and in circumferential joint, longitudinal stress is developed.



If A is the gross area and A_{eff} is the effective resisting area then,

$$\text{Efficiency} = A_{\text{eff}}/A$$

$$\text{Bursting force} = p L d$$

$$\text{Resisting force} = \sigma_c \times A_{\text{eff}} = \sigma_c \times \eta_L \times A = \sigma_c \times \eta_L \times 2 t L$$

Where η_L = Efficiency of Longitudinal joint

$$\text{Bursting force} = \text{Resisting force}$$

$$p L d = \sigma_c \times \eta_L \times 2 t L$$

$$\sigma_c = \frac{p \times d}{2 \times t \times \eta_L} \dots\dots\dots(1)$$

If η_c = Efficiency of circumferential joint

$$\text{Efficiency} = A_{\text{eff}}/A$$

$$\text{Bursting force} = (\pi d^2/4)p$$

$$\text{Resisting force} = \sigma_L \times A'_{\text{eff}} = \sigma_L \times \eta_c \times A' = \sigma_L \times \eta_c \times \pi d t$$

Where η_L = Efficiency of circumferential joint

$$\text{Bursting force} = \text{Resisting force}$$

$$\sigma_L = \frac{p \times d}{4 \times t \times \eta_c} \dots\dots\dots(2)$$

A cylindrical tank of 750mm internal diameter, 12mm thickness and 1.5m length is completely filled with an oil of specific weight 7.85 kN/m³ at atmospheric pressure. If the efficiency of longitudinal joints is 75% and that of circumferential joints is 45%, find the pressure head of oil in the tank. Also calculate the change in volume. Take permissible tensile stress of tank plate as 120 MPa and $E = 200$ GPa, and $\mu = 0.3$.

SOLUTION:

Let p = max permissible pressure in the tank.

Then we have, $\sigma_L = (p \times d) / (4 \times t) \eta_C$

$$120 = (p \times 750) / (4 \times 12) 0.45$$

$$p = 3.456 \text{ MPa.}$$

Also, $\sigma_C = (p \times d) / (2 \times t) \eta_L$

$$120 = (p \times 750) / (2 \times 12) 0.75$$

$$p = 2.88 \text{ MPa.}$$

Max permissible pressure in the tank, $p = 2.88 \text{ MPa}$.

$$\text{Vol. Strain, } \frac{dv}{V} = \frac{(p \times d)}{(4 \times t \times E)} \times (5 - 4 \times \mu)$$

$$= \frac{(2.88 \times 750)}{(4 \times 12 \times 200 \times 10^3)} \times (5 - 4 \times 0.3) = 8.55 \times 10^{-4}$$

$$dv = 8.55 \times 10^{-4} \times V = 8.55 \times 10^{-4} \times \frac{\pi}{4} \times 750^2 \times 1500 = 0.567 \times 10^6 \text{ mm}^3.$$

$$= 0.567 \times 10^{-3} \text{ m}^3 = \underline{0.567} \text{ litres.}$$

A boiler shell is to be made of 15mm thick plate having a limiting tensile stress of 120 N/mm². If the efficiencies of the longitudinal and circumferential joints are 70% and 30% respectively determine;

i) The maximum permissible diameter of the shell for an internal pressure of 2 N/mm².

(ii) Permissible intensity of internal pressure when the shell diameter is 1.5m.

SOLUTION:

(i) To find the maximum permissible diameter of the shell for an internal pressure of 2 N/mm²:

a) Let limiting tensile stress = Circumferential stress = $\sigma_c = 120 \text{ N/mm}^2$.

$$\text{i. e., } \sigma_c = \frac{p \times d}{2 \times t \times \eta_L}$$

$$120 = \frac{2 \times d}{2 \times 15 \times 0.7}$$

$$d = 1260 \text{ mm}$$

b) Let limiting tensile stress = Longitudinal stress = $\sigma_L = 120 \text{ N/mm}^2$.

$$\text{i. e., } \sigma_L = \frac{p \times d}{4 \times t \times \eta_c}$$

$$120 = \frac{2 \times d}{4 \times 15 \times 0.3} \quad . \quad d = 1080 \text{ mm}$$

The maximum diameter of the cylinder in order to satisfy both the conditions = 1080 mm.

(ii) To find the permissible pressure for an internal diameter of 1.5m:
(d=1.5m=1500mm)

a) Let limiting tensile stress = Circumferential stress = $\sigma_c = 120\text{N/mm}^2$.

$$\text{i. e., } \sigma_c = \frac{p \times d}{2 \times t \times \eta_L}$$

$$120 = \frac{p \times 1500}{2 \times 15 \times 0.7}$$

$$p = 1.68 \text{ N/mm}^2.$$

b) Let limiting tensile stress = Longitudinal stress = $\sigma_L = 120\text{N/mm}^2$.

$$\text{i. e., } \sigma_L = \frac{p \times d}{4 \times t \times \eta_C}$$

$$120 = \frac{p \times 1500}{4 \times 15 \times 0.3}$$

$$p = 1.44 \text{ N/mm}^2.$$

The maximum permissible pressure = 1.44 N/mm².

PROBLEMS FOR PRACTICE

PROBLEM 1:

Calculate the circumferential and longitudinal strains for a boiler of 1000mm diameter when it is subjected to an internal pressure of 1MPa. The wall thickness is such that the safe maximum tensile stress in the boiler material is 35 MPa. Take $E=200\text{GPa}$ and $\mu=0.25$.

(Ans: $\varepsilon_C=0.0001531$, $\varepsilon_L=0.00004375$)

PROBLEM 2:

A water main 1m in diameter contains water at a pressure head of 120m. Find the thickness of the metal if the working stress in the pipe metal is 30 MPa. Take unit weight of water = 10 kN/m^3 .

(Ans: $t=20\text{mm}$)

PROBLEM 3:

A gravity main 2m in diameter and 15mm in thickness. It is subjected to an internal fluid pressure of 1.5 MPa. Calculate the hoop and longitudinal stresses induced in the pipe material. If a factor of safety 4 was used in the design, what is the ultimate tensile stress in the pipe material?

(Ans: $\sigma_C=100$ MPa, $\sigma_L=50$ MPa, $\sigma_U=400$ MPa)

PROBLEM 4:

At a point in a thin cylinder subjected to internal fluid pressure, the value of hoop strain is 600×10^{-4} (tensile). Compute hoop and longitudinal stresses. How much is the percentage change in the volume of the cylinder? Take $E=200$ GPa and $\mu=0.2857$.

(Ans: $\sigma_C=140$ MPa, $\sigma_L=70$ MPa, %age change=0.135%.)

PROBLEM 5:

A cylindrical tank of 750mm internal diameter and 1.5m long is to be filled with an oil of specific weight 7.85 kN/m^3 under a pressure head of 365 m. If the longitudinal joint efficiency is 75% and circumferential joint efficiency is 40%, find the thickness of the tank required. Also calculate the error of calculation in the quantity of oil in the tank if the volumetric strain of the tank is neglected. Take permissible tensile stress as 120 MPa, $E=200\text{GPa}$ and $\mu=0.3$ for the tank material. (Ans: $t=12 \text{ mm}$, error=0.085%.)

Assignment # 1: Thin Walled Cylinders Problems

47

Solve all the problems given in “Problem for Practice”.

Also try to comprehend the reality of the attained answers.

Due Date: 11th February, 2020.

Thank-you for Listening!

48

Believe, solution exists. (Shad)

⌘Matter-Antimatter Theory!

⌘Theory of Couples!

⌘Problem-Solution Theory!